Solutions

2.6 and 2.7

Objectives:

- Word Problems

Simple Interest: (Recall: The simple interest formula is $I = PRT$ where I is interest, P is principal, R is rate and T is years.) Ronelio invests \$20,000 in two one-year certificates of deposit. One certificate pays 3%, and the other pays $3\frac{3}{4}\%$ simple interest annually.

1. Construct a model for the total interest Ronelio earns in one year on his investments. (Let x represent the amount invested.)

2. If Ronelio's total interest is \$697.50, how much money did he invest in each certificate.

1. If x is amount invested in certificate one then $20{,}000 - x$ is invested in certificate two

So $I(x) = x(0.03)\cdot 1 + (20{,}000 - x)(0.0375)1$.

2. $697.50 = 0.03x + (20{,}000 - x)(0.0375)$

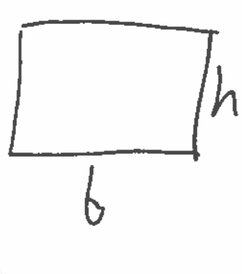
$= 750 - 0.0075x$

$-52.5 = -0.0075x$

$x = \$7{,}000$ in certificate one and \$13,000 in certificate two

Recall: A equation of the form $f(x) = ax^2 + bx + c$ is the equation of a parabola. A parabola has either a minimum or a maximum.

Optimization Word Problem: A farmer wants to fence in his lettuce crop. He has 260ft of fencing and wants to maximize the amount of crop he can fence around in a rectangular fashion. Find the dimensions (length and width) that will maximize the fenced in area.



$A = bh \qquad P = 2b + 2h = 260$

So $h = \dfrac{260 - 2b}{2} = 130 - b$

So $A = b(130 - b) = -b^2 + 130b$.

Max Area is at $\dfrac{-b}{2a} = \dfrac{-130}{-2} = 65$. So $h = b = 65$ft.

Mixture Word Problem: Lulu has one trail mix that is 2% chocolate candy. She has another trail mix that is 5% chocolate candy. If she has 4lbs of the 2% trail mix, how many pounds of trail mix two must she add to have trail mix that is 4% chocolate candy?

Amount of chocolate = (lbs of trail mix) · % of chocolate.

So $(x+4)\cdot(0.04) = 4(0.02) + x(0.05)$

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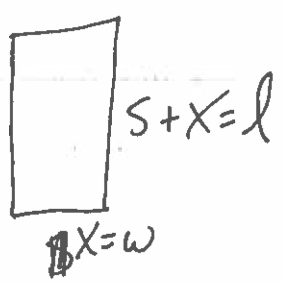
$0.04x + 0.16$

So $0.01x = 0.08$ and thus $x = 8$ lbs of second trail mix.

Geometry: A graphic artist needs to construct a design that uses a rectangle whose length is 5cm longer than its width x.

1. Construct a model that gives the perimeter of the rectangle.
2. If the perimeter of the rectangle is 26cm, what are the dimensions of the rectangle?



1. $P = 2l + 2w = 2(5+x) + 2x$
$= 10 + 4x$

2. $26 = 10 + 4x \Rightarrow 16 = 4x \Rightarrow x = 4$cm.

So width is 4cm and length is 9cm.

Objectives:

- Intersection of two Lines
- Applications

The intersection of two lines. If you have have two lines and want to find where they intersect, you set them equal to each other. Parallel lines never meet.

Example: Find the intersection of $y = 2x - 3$ and $2y + 3x = 4$.

$$Y = \frac{4-3x}{2}$$

(1) Set them equal.

$$2x-3 = \frac{4-3x}{2} \iff 4x-6 = 4-3x \iff 7x = 10 \iff x = \frac{10}{7}$$

(2) Plug in x. $y = 2\left(\frac{10}{7}\right) - 3 = -\frac{1}{7}$. So they meet at $\left(\frac{10}{7}, -\frac{1}{7}\right)$.

Break-Even Point (The break-even point is the point where the cost and profit(savings) are equal.) Lina is considering installing solar panels on her house. Solar Advantage offers to install solar panels that generate 320kWh of electricity per month for an installation fee of \$15,000. She uses 350kWh of electricity per month, and her local utility company charges \$0.20 per kWh.

1. If Lina gets all her electrical power from the local utility company, find a linear function U that models the cost $U(x)$ of electricity for x months of service.
2. If Lina has Solar Advantage install solar panels on her roof that generate 320kWh of power per month, find a linear function S that models the cost $S(x)$ of electricity for x months of service.
3. Determine the number of months it would take to reach the break-even point for installation of solar panels.

1. $U(x) = 350 \cdot (0.20) \cdot x = 70x$

2. Still needs 30kWh from utility company, costing $30(0.20) = \$6$ a month.

So $S(x) = 6x + 15{,}000$.

3. $U(x) = S(x)$ when $70x = 6x + 15{,}000 \iff 64x = 15{,}000 \iff x = 234.375$ months

Catching Up: Kumar leaves his house at 7:30 A.M. and cycles to school. Kumar's mother notices that he has left his lunch at home. She leaves the house by car 5 minutes after Kumar left to give him his lunch. Kumar cycles at an average speed of 8 mi/h, and his mother drives at an average speed of 24 mi/h.

1. Find a linear equation that models the distance y Kumar's mother has traveled, x hours after his mother has left home.
2. Find a linear equation that models the distance y Kumar has traveled x hours after his mother has left home.
3. Determine how long it takes Kumar's mother to catch up with Kumar. How far have they traveled at the time they meet?

1. $y = 24x$

2. $y = 8x + \frac{2}{3}$

~~The~~ When his mother left home, Kumar had already traveled $8\,\text{mi/h} \cdot \left(\frac{1}{12}\,h\right) = \frac{2}{3}$ miles.

3. Set them equal $24x = 8x + \frac{2}{3}$

$16x = \frac{2}{3} \Leftrightarrow x = \frac{1}{24}$ hours $= 2.5$min

Plug in x $24\left(\frac{1}{24}\right) = 1$ mile.

Supply and Demand: An economist models the market for corn by the following equations.

- Supply: $y = 4.18p - 11.5$
- Demand: $y = -1.06p + 19.3$

Here, p is the price per bushel (in dollars), and y is the number of bushels produced and sold (in billions).

1. Use the model for supply to determine at what point the price is so low that no corn is produced.
2. Use the model for demand to determine at what point the price is so high that no corn is sold.
3. Find the equilibrium price and the quantities that are produced and sold at equilibrium.

1. $y=0$ no corn is produced.

$0 = 4.18p - 11.5 \Rightarrow \frac{11.5}{4.18} = p = \$2.75$

2. $y=0$ no corn is sold

$0 = -1.06p + 19.3 \Rightarrow \frac{-19.3}{-1.06} = p = \$18.20$

3. Set them equal $4.18p - 11.5 = -1.06p + 19.3$

$5.24p = 30.8$

$p = \frac{30.8}{5.24} = \$5.88$